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7. Brief review of the theory: pneumatic calculations

7.1. Pneumatic assumptions

The submodels in the **Pneumatic** library use the first and second law of thermodynamics.

The following assumptions are used in the submodels of the **Pneumatic** library:

- the velocity of gas is not supersonic,
- the flow is one-dimensional,
- the gas is considered to be a perfect, semi-perfect, or a real gas,
- the gravitational effect on the gas is not taken into account,
- the intensive variables, i.e. pressure and temperature, are assumed to be average values in the whole volume considered (lumped modeling),
- the kinetic energy is ignored in chamber submodels,
- reversible thermodynamic transformations are used in the orifice submodels.

7.2. Modeling pneumatic orifices

Almost all pneumatic systems you are likely to model will contain orifices. This section describes how they are modeled. The kinetic energy of the upstream flow is ignored. The mass flow rate dm through a pneumatic orifice is a function of input pressures (absolutes) and temperatures.

Assuming the gas is a compressible fluid, we have:

$$(7.1) \quad dm = A \cdot C_q \cdot C_m \cdot \frac{P_{up}}{\sqrt{T_{up}}} \quad \text{$$

where T_{up} is the upstream temperature and C_m is the flow parameter.

The flow coefficient C_q is introduced to adjust the theoretical relationship to experiments. This coefficient is used to include extra losses due to local friction and loss of kinetic energy. It may represent frictional drag in pipes for instance, or local resistance due to changes in geometry or in flow direction.

The flow parameter is determined using the following general expression:

$$(7.2) \quad C_m = \sqrt{\frac{2}{1-\gamma_s} \cdot \frac{T_{up}}{v_{up} \cdot P_{up}}} \cdot \sqrt{\left(\frac{P_{dn}}{P_{up}}\right)^{2\gamma_s} - \left(\frac{P_{dn}}{P_{up}}\right)^{1+\gamma_s}}$$

where T_{up} , P_{up} and v_{up} are respectively the upstream temperature, pressure and specific volume, T_{dn} is the downstream temperature and γ_s the isentropic calorific factor.

Considering a perfect or semi-perfect (ideal) gas behavior, the flow parameters written as following:

$$(7.3) \quad C_m = \begin{cases} \sqrt{\frac{2\gamma}{r(\gamma-1)}} \cdot \sqrt{\left(\frac{P_{dn}}{P_{up}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{dn}}{P_{up}}\right)^{\frac{\gamma+1}{\gamma}}} & \text{if } \frac{P_{dn}}{P_{up}} > P_{cr} \text{ (subsonic)} \\ \sqrt{\frac{\gamma}{r} \cdot \left(\frac{2}{\gamma-1}\right)^{\frac{\gamma+1}{\gamma-1}}} = \sqrt{\frac{2\gamma}{r(\gamma-1)}} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} & \text{if } \frac{P_{dn}}{P_{up}} \leq P_{cr} \text{ (sonic)} \end{cases}$$

Therefore, when the flow is sonic, the flow parameter is constant and only a function of the gas properties, whereas in subsonic flow, the flow parameter is a function of the pressure ratio $\frac{P_{dn}}{P_{up}}$.

The critical pressure ratio at which the flow switches from sonic to subsonic mode can be calculated using the equation:

$$(7.4) \quad P_{cr} = \left(\frac{2 \cdot \gamma_s}{\gamma_s + 1}\right)^{\frac{1}{1-\gamma_s}}$$

Here again, considering a perfect or semi-perfect gas behavior, the previous equation becomes the well-known critical pressure ratio expressed as a function of the specific heat ratio:

$$(7.5) \quad P_{cr} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

The evolution of the flow parameter for air is shown in [Fig. 7.1](#).

$$T_{axialL} = f_2 F_a D_m$$

Fig. 7.1 Flow parameter as a function of pressure ratio [1](#)

The enthalpy flow rate dmh is calculated as follows:

$$(7.6) \quad dmh = dm \cdot h(P_{up}, T_{up})$$

where $h(P_{up}, T_{up})$ is the gas specific enthalpy at upstream conditions.

The specific enthalpy derives from the polynomial form of the specific heat of the gas:

$$(7.7) \quad dh = C_p \cdot dT$$

with:

$$(7.8) \quad C_p = C_{p0} \cdot [C_{pc} + C_{pt} \cdot \Delta T + C_{pt2} \cdot \Delta T^2]$$

After integration:

$$(7.9) \quad h = C_{pc} \cdot C_{p0} \cdot T_0 + C_{p0} \cdot \Delta T \cdot \left(C_{pc} + C_{pt} \cdot \frac{\Delta T}{2} + C_{pt2} \cdot \frac{\Delta T^2}{3} \right)$$

The *vena contracta* mean gas velocity is calculated from the mass flow rate, the cross-sectional area of the fitting and the flow coefficient C_q using the assumption that the evolution is isentropic (adiabatic and reversible) between the upstream port and the *vena contracta*.

Therefore, the *vena contracta* specific volume (or density) is a function of the upstream specific volume (or density) independently of the gas behavior:

$$(7.10) \quad v_{vc} = \left(\frac{2 \cdot \gamma_s}{\gamma_s + 1} \right)^{\frac{\gamma_s}{1-\gamma_s}} \cdot v_{up}$$

where v_{vc} is the *vena contracta* specific volume and v_{up} is the upstream specific volume.

The critical gas velocity corresponds to the speed of sound of the gas calculated at the *vena contracta* temperature:

$$(7.11) \quad v_{max} = \sqrt{\frac{2}{1 + \gamma_s} \cdot v_{up} \cdot P_{up}}$$

where v_{max} is the critical gas velocity, v_{up} is the upstream specific volume, P_{up} is the upstream pressure and γ_s the isentropic calorific factor.

Considering a perfect or semi-perfect gas behavior, the previous expression of the critical velocity becomes:

$$(7.12) \quad v_{\max} = \sqrt{\frac{2}{\gamma + 1} \cdot \gamma \cdot r \cdot T_{\text{up}}}$$

where T_{up} is the upstream temperature, r is the perfect gas constant and γ the specific heat ratio.

This is equivalent to the critical velocity computed at the vena contracta temperature:

$$(7.13) \quad v_{\max} = \sqrt{\gamma \cdot r \cdot T_{\text{vc}}}$$

where $v_{\max}$ is the critical gas velocity, T_{vc} is the *vena contracta* temperature, r is the perfect gas constant and γ the specific heat ratio.

7.2.1. Submodel with fixed flow coefficient

A straightforward approximation consists in taking the flow coefficient C_q as a constant.

7.2.2. Submodel with polynomial Perry's flow coefficient

Since the flow aspect changes with the pressure ratio, a slightly more complex hypothesis is to assume that the flow coefficient C_q is a polynomial function of the pressure ratio $\frac{P_{\text{dn}}}{P_{\text{up}}}$.

The Perry's flow coefficient is used for sharp-edged orifices and it can be approximated by a polynomial function of the pressure ratio $\frac{P_{\text{dn}}}{P_{\text{up}}}$:

$$(7.14) \quad C_q = \left[\left[\left(-1.6827 \cdot \frac{P_{\text{dn}}}{P_{\text{up}}} + 4.6 \right) \cdot \frac{P_{\text{dn}}}{P_{\text{up}}} - 3.9 \right] \cdot \frac{P_{\text{dn}}}{P_{\text{up}}} + 0.8415 \right] \cdot \frac{P_{\text{dn}}}{P_{\text{up}}} - 0.1 \right] \cdot \frac{P_{\text{dn}}}{P_{\text{up}}} + 0.8414$$

The variation of flow coefficient C_q is shown in [Fig. 7.2](#).

$$T_{\text{radLoad1}} = C_{\text{oeff}} f_1 (P_0 + P_1) D_m$$

Fig. 7.2 Perry's flow coefficient

7.2.3. Submodel with ISO-6358 approximation

When some experimental measurements for an orifice are provided, it is more common to use the ISO 6358 norm (see [\[4\]](#)) which defines the static characteristics of a pneumatic orifice using two parameters only:

- The **critical pressure ratio b** is the pressure ratio which localizes the transition between subsonic and sonic flow regions.

- The **sonic conductance C** is the flow characteristics of the orifice (in $\text{s.m}^4/\text{kg}$).

In the ISO 6358 norm, the mass flow rate is evaluated using the following equations:

$$(7.15) \quad dm = \begin{cases} C \cdot P_{\text{up}} \cdot \rho_0 \cdot \sqrt{\frac{T_0}{T_{\text{up}}}} \sqrt{1 - \left(\frac{\frac{P_{\text{dn}}}{P_{\text{up}}} - b}{1-b} \right)^2} & \text{if } \frac{P_{\text{dn}}}{P_{\text{up}}} > b \text{ (subsonic)} \\ C \cdot P_{\text{up}} \cdot \rho_0 \cdot \sqrt{\frac{T_0}{T_{\text{up}}}} & \text{if } \frac{P_{\text{dn}}}{P_{\text{up}}} \leq b \text{ (sonic)} \end{cases}$$

Where ρ_0 is the density of the gas at reference conditions: $T_0 = 293.15 \text{ K}$, $P_0 = 10^5 \text{ Pa}$ and 65 % relative humidity

In this case, the *vena contracta* mean gas velocity cannot be estimated since the *vena contracta* density is unknown (no information is available on the evolution between upstream and *vena contracta* conditions, the equivalent flow area and the flow coefficient being unknown).

7.3. Modeling pneumatic chambers [1](#)

The pneumatic chamber is the second main submodel used in the simulation of pneumatic systems. In this lumped parameter element, we calculate the temperature and the pressure which are average values in the control volume.

The temperature and pressure are state variables since they are computed from their time derivatives.

7.3.1. Mass balance [1](#)

The mass variation can also be written as below:

$$(7.16) \quad \frac{dm}{dt} = \frac{d}{dt} (\rho \cdot V) = \rho \cdot \frac{dV}{dt} + V \cdot \frac{d\rho}{dt} = \rho \cdot \frac{dV}{dt} + V \cdot \left[\left(\frac{\partial \rho}{\partial P} \right)_T \cdot \frac{dP}{dt} + \left(\frac{\partial \rho}{\partial T} \right)_P \cdot \frac{dT}{dt} \right]$$

where:

- $\frac{dm}{dt}$ is the mass variation in the volume [kg/s],
- V is the volume [m^3],
- ρ is the gas density [kg / m^3].

7.3.2. Energy balance [1](#)

The energy in the control volume is given by:

$$(7.17) \quad E = m \cdot u + \frac{mV^2}{2} + mgz$$

These three terms represent the internal energy, the kinetic energy and the potential energy respectively.

We simplify the previous equation introducing the assumption that **the kinetic and potential energies in the control volume are ignored** (see [appendix E](#)). This leads to:

$$(7.18) \quad E = mu = U$$

where u is the internal energy by mass unit, and U is the total internal energy.

We can express the variation of internal energy U using the first law of thermodynamics applied to an open system:

$$(7.19) \quad dU = \sum dm_i \cdot h_i + \delta Q + dW_{\text{ext}}$$

where $dm_i \cdot h_i$ is the incoming total enthalpy flow at port i during time dt , obtained by the product of the incoming mass flow during time dt and the specific enthalpy at port i , δQ is the heat exchanged through the control volume during dt and dW_{ext} is the work variation of the system.

The external work supplied by the system is due to the mechanical displacement of a piston connected to the volume, and therefore:

$$(7.20) \quad \frac{dW_{\text{ext}}}{dt} = -p \frac{dV}{dt}$$

where $\frac{dV}{dt}$ is the volume variation due to a piston displacement.

From the relation of internal energy U :

$$(7.21) \quad U = m \cdot u$$

The energy variation can also be written as follows:

$$(7.22) \quad dU = d(m \cdot u) = dm \cdot u + m \cdot du = \sum_i dm_i \cdot u + m \cdot du$$

Introducing the enthalpy $h = u + P \cdot v$ and using [Eq.7.19](#) and [Eq.7.20](#) leads to:

$$(7.23) \quad m \cdot dh - V \cdot dP = \sum_i dm_i \cdot h_i - \sum_i dm_i \cdot h + \delta Q$$

Considering variation over time, this leads to:

$$(7.24) \quad \left(m \cdot \left(\frac{\partial h}{\partial P} \right)_T - V \right) \cdot \frac{dP}{dt} + m \cdot \left(\frac{\partial h}{\partial T} \right)_P \cdot \frac{dT}{dt} = \sum_i \frac{dm_i}{dt} \cdot h_i - \sum_i \frac{dm_i}{dt} \cdot h + \delta Q$$

Temperature and pressure time derivatives are computed solving the following system composed by [Eq.7.16](#) and [Eq.7.24](#):

$$(7.25) \quad \begin{bmatrix} V \cdot \left(\frac{\partial \rho}{\partial P} \right)_T & V \cdot \left(\frac{\partial \rho}{\partial T} \right)_P \\ m \cdot \left(\frac{\partial h}{\partial P} \right)_T - V & m \cdot \left(\frac{\partial h}{\partial T} \right)_P \end{bmatrix} \cdot \begin{bmatrix} \frac{dP}{dt} \\ \frac{dT}{dt} \end{bmatrix} = \begin{bmatrix} \sum_i \frac{dm_i}{dt} - \rho \cdot \frac{dV}{dt} \\ \sum_i \frac{dm_i}{dt} \cdot h_i - \sum_i \frac{dm_i}{dt} \cdot h + \delta Q \end{bmatrix}$$

Note

All the equations above are available for any equation of state.

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