

Utility name : pn2rcqfix

Purpose

pn2rcqfix computes the mass flow rate and the enthalpy flow rate through an orifice with a fixed flow discharge coefficient (C_q) .

Description

The mass flow rate of any compressible fluid through an orifice may be written [\[1, p.80\]](#):

$$\dot{m} = A \cdot C_q \cdot C_m \cdot \frac{p_{\mathrm{up}}}{\sqrt{T_{\mathrm{up}}}}$$

where (A) is the orifice area, (C_q) the flow coefficient, (C_m) the flow parameter, (p_{up}) the upstream absolute pressure and (T_{up}) the upstream temperature.

The flow coefficient (C_q) is introduced to adjust the theoretical relation to experiments. This coefficient is less than one and is used to include extra losses due to local friction and loss of kinetic energy.

A straightforward approximation consists in taking the flow coefficient (C_q) as a constant, which is an input parameter of pn2rcqfix.

The type of the flow (subsonic or sonic) depends on the pressure ratio $\left(\frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}}\right)$, the critical pressure ratio (p_{cr}) [null] and the gas assumption.

In all cases, the enthalpy flow rate $(\dot{h})$ [J/s] is calculated as follows:

$$\dot{h} = \dot{m} \cdot h(p_{\mathrm{up}}, T_{\mathrm{up}})$$

where (h) is the upstream enthalpy [J/kg]. Please refer to the utility [pn2hpti](#) for detailed computations.

Perfect gas assumption

Assuming a perfect gas behavior, the critical pressure ratio is computed as [\[1, p.80\]](#)[\[2\]](#):

$$p_{\mathrm{cr}} = \left(\frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}}\right)_{\mathrm{cr}} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

where (γ) is the specific heat ratio:

$$\gamma = \frac{c_p}{c_v}$$

For details about its computation, refer to the utility [pn2gpti](#).

The mass flow parameter may be written as the following for a perfect gas [\[1, p.80\]](#):

$$C_m = \left[\sqrt{\frac{2}{\gamma}} \frac{p_{dn}}{p_{up}} \right]^{\frac{2}{\gamma}} - \left[\frac{p_{dn}}{p_{up}} \right]^{\frac{\gamma+1}{\gamma}} \quad \text{if } \frac{p_{dn}}{p_{up}} > p_{cr} \quad \text{(subsonic)}$$

$$\sqrt{\frac{2}{\gamma}} \frac{p_{dn}}{p_{up}} \left[\frac{2}{\gamma+1} \right]^{\frac{1}{\gamma-1}} \quad \text{if } \frac{p_{dn}}{p_{up}} \leq p_{cr} \quad \text{(sonic)}$$

where:

- γ is the specific heat ratio [null],
- T_{up} is the upstream temperature [K],
- ρ_{up} is the upstream density [kg/m³],
- p_{up} is the (total) upstream pressure [PaA],
- p_{dn} is the (total) downstream pressure [PaA],
- and r is the specific gas constant [J/kg/K] given by utility [pn2ri](#).

It is important to note that in many books instead of p_{dn} in formulas above generally the static pressure at vena contracta p_{vc} is used. However this pressure is hardly accessible and some authors readily replace it by the total pressure p_{dn} ; to account for this change they introduce the *flow coefficient* C_q used above in place of the *discharge coefficient* C_d . In other words C_q allows to model irreversible losses occurring after the vena contracta. The reader may refer for instance to [\[4\]](#).

On the figure below is shown the agreement between Amesim and the literature for the mass flow parameter C_m (case of air) [\[1, p.80\]](#).

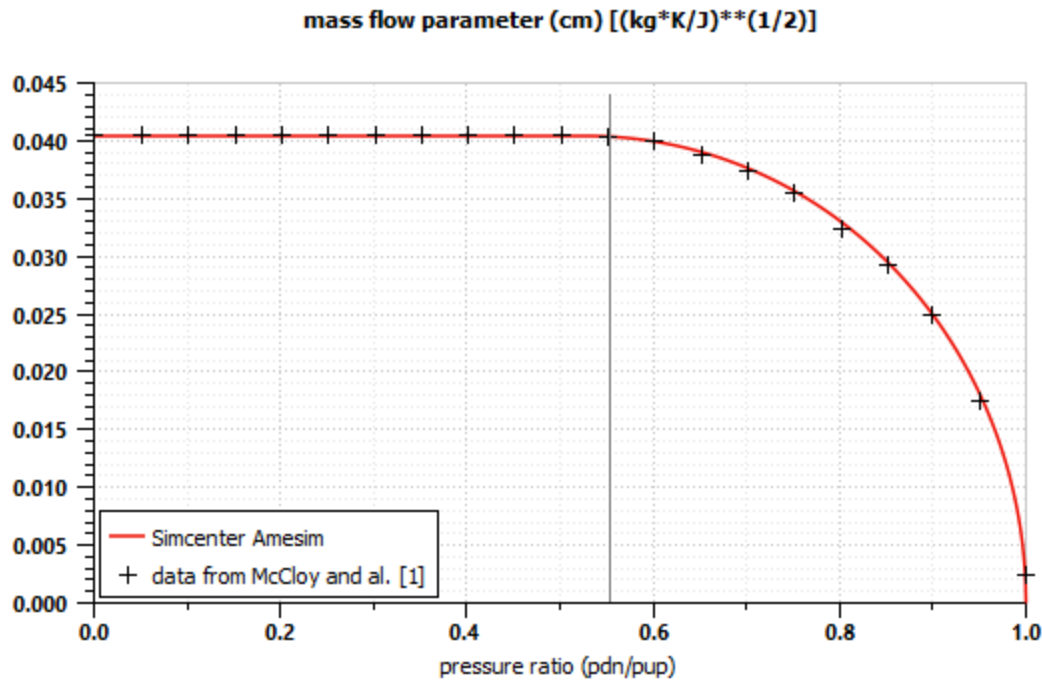


Figure 1: mass flow parameter (C_m) as a function of the pressure ratio (p_{dn}/p_{up}) for air

The vertical line indicates the critical pressure, which is equal to 0.528 for air. On the left side of this line, the flow is sonic, C_m is constant and the mass flow rate is a function of only the upstream pressure and temperature. On the right side of the line, the flow is subsonic and the mass flow rate is also a function of the downstream pressure.

The gas velocity at vena contracta is determined from the upstream conditions, considering an isentropic evolution through the restriction [2, p.67]:

$$v_c = \begin{cases} \sqrt{\frac{2}{\gamma-1} \frac{p_{up}}{\rho_{up}}} & \text{if } \frac{p_{dn}}{p_{up}} \geq \left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \\ \sqrt{\frac{2}{\gamma+1} \frac{p_{up}}{\rho_{up}}} & \text{if } \frac{p_{dn}}{p_{up}} < \left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \end{cases}$$

Real gas

The mathematical relations given above are correct only for a perfect gas; for a real gas, or even a semi-perfect gas, they become approximations. However, one author generalizes the use of these equations to real gases [3, p.348-349][5, p.110], by using the fact that the isentropic calorific factor:

$$\left[\gamma_s = \frac{p}{\rho} \left(\frac{\partial \rho}{\partial p} \right)_s \right]$$

is equal to $(1/\gamma)$ for a perfect gas (see [pn2gspti](#)). Thus as an approximation $(1/\gamma)$ may be replaced by $(1/\gamma_s)$ in the previous formulas. The critical pressure ratio is approximated as

$$p_{\mathrm{cr}} = \left(\frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} \right)_{\mathrm{cr}} = \left(\frac{2 \gamma_s}{\gamma_s + 1} \right)^{\frac{1}{1 - \gamma_s}}$$

The mass flow parameter (C_m) is computed using

$$C_m = \left[\begin{array}{l} \sqrt{\frac{2}{1 - \gamma_s}} \cdot \frac{\rho_{\mathrm{up}} T_{\mathrm{up}}}{p_{\mathrm{up}}} \cdot \sqrt{\left(\frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} \right)^{2 \gamma_s} - \left(\frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} \right)^{1 + \gamma_s}} \quad \text{if} \quad \frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} > p_{\mathrm{cr}} \quad \text{(subsonic)} \\ \sqrt{\frac{2}{1 + \gamma_s}} \cdot \frac{\rho_{\mathrm{up}} T_{\mathrm{up}}}{p_{\mathrm{up}}} \cdot \left(\frac{2 \gamma_s}{\gamma_s + 1} \right)^{\frac{\gamma_s}{1 - \gamma_s}} \quad \text{if} \quad \frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} \leq p_{\mathrm{cr}} \quad \text{(sonic)} \end{array} \right]$$

and the gas velocity at vena contracta [m/s] is:

$$v_c = \left[\begin{array}{l} \sqrt{\frac{2}{1 - \gamma_s}} \cdot \frac{p_{\mathrm{up}}}{\rho_{\mathrm{up}}} \cdot \sqrt{1 - \left(\frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} \right)^{1 - \gamma_s}} \quad \text{if} \quad \frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} > p_{\mathrm{cr}} \quad \text{(subsonic)} \\ \sqrt{\frac{2}{1 + \gamma_s}} \cdot \frac{p_{\mathrm{up}}}{\rho_{\mathrm{up}}} \cdot \left(\frac{2 \gamma_s}{\gamma_s + 1} \right)^{\frac{\gamma_s}{1 - \gamma_s}} \quad \text{if} \quad \frac{p_{\mathrm{dn}}}{p_{\mathrm{up}}} \leq p_{\mathrm{cr}} \quad \text{(sonic)} \end{array} \right]$$

Laminar transition

When the pressure ratio approaches 1, the flow is laminar. The transition between turbulent and laminar is set by the *accuracy* parameter defined in [PNAC001](#) submodel. By default, the pressure ratio accuracy is set to 0.9999.

Note that a specific smoothing law for the mass flow parameter (C_m) is implemented in laminar region to ensure the flow continuity. This law differs from the equations above.

C call prototype

```
extern void pn2rcqfix_
(
    double *dhl, double *dml, double *templ, double *pressl,
```

```
double *temp2, double *press2, double *area, double *cq,  
int *gi, double *cm, double *gasvel, int *reg1  
);
```

Arguments

Inputs

- **temp1**, temperature at port 1 [K]
- **press1**, pressure at port 1 (absolute) [PaA]
- **temp2**, temperature at port 2 [K]
- **press2**, pressure at port 2 (absolute) [PaA]
- **area**, orifice area [m²]
- **cq**, discharge coefficient [null]
- **gi**, gas type index

Outputs

- **dh1**, enthalpy flow rate at port 1 [J/s]
- **dm1**, mass flow rate at port 1 [kg/s]
- **cm**, mass flow parameter [(kg.K/J)^{1/2}]
- **gasvel**, gas velocity at vena contracta (sign of dm1) [m/s]
- **reg1**, flag for flow direction

See also

[PNOR001](#)

References

- [1] D.McCloy, H. Martin: "Control of Fluid Power: Analysis and design", 2nd edition. Ellis Horwood Limited 1980.
- [2] Blackburn J.F., G. Reethof and J.L. Shearer, "Fluid Power Control", M.I.T. Press, 1960.
- [3] Borel, Lucien and Favrat, Daniel. "Thermodynamics and Energy Systems Analysis". First edition. ISBN 978-2-940222-45-2. EPFL Press. 2010.
- [4] McCloy, D., Beck, A.: Some Cavitation Effects in Spool Valve Orifices , Proc Instn Mech Engrs 182(8), 1967-68.
- [5] Borel, Lucien, Favrat, Daniel, Nguyen, Dinh Lan, Batato, Magdi: Thermodynamics and Energy Systems Analysis , Solved Problems and Exercises , EPFL Press, 2012.